LESSON 2: PERFORMANCE OF CONTROL SYSTEMS

ET 438a

Automatic Control Systems Technology

LEARNING OBJECTIVES

After this presentation you will be able to:

- Explain what constitutes good control system performance.
- Identify controlled, uncontrolled, and unstable control system response.
- Analyze measurement error in measurement sensors.
- Determine sensor response.
- Apply significant digits and basic statistics to analyze measurements.

CONTROL SYSTEM PERFORMANCE

System control variable changes over time so error changes with time.

$$E(t) = R - C(t)$$

Where $E(t)$ = error as a function of time
R = setpoint (reference) value
$C(t)$ = control variable as a function of time

Determine performance criteria for adequate control system performance. System should maintain desired output as closely as possible when subjected to disturbances and other changes

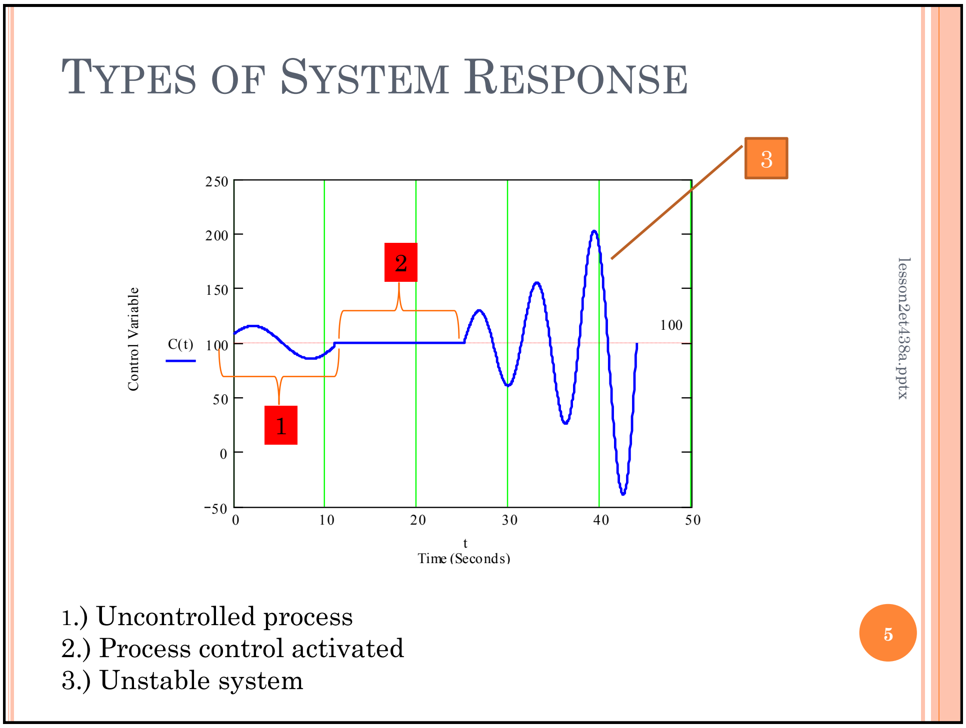
CONTROL SYSTEM OBJECTIVES

1.) System error minimized. $E(t) = 0$ after changes or disturbances after some finite time.

2.) Control variable, $c(t)$, stable after changes or disturbances after some finite time interval

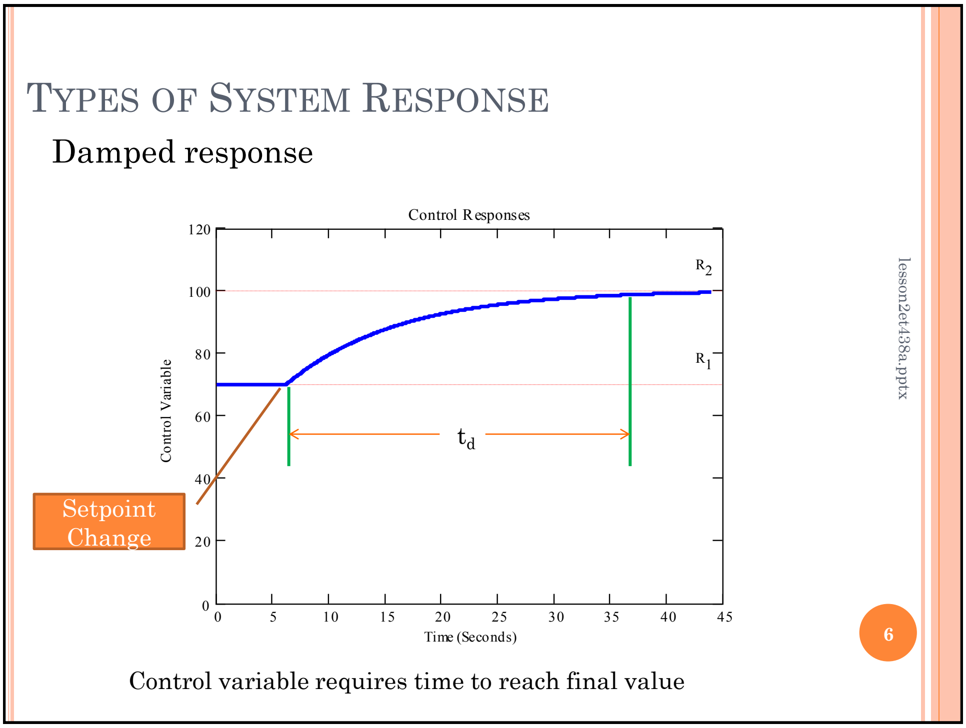
Stability Types

Steady-state regulation : $E(t) = 0$ or within tolerances
Transient regulation - how does system perform under change in reference (tracking)



TYPES OF SYSTEM RESPONSE

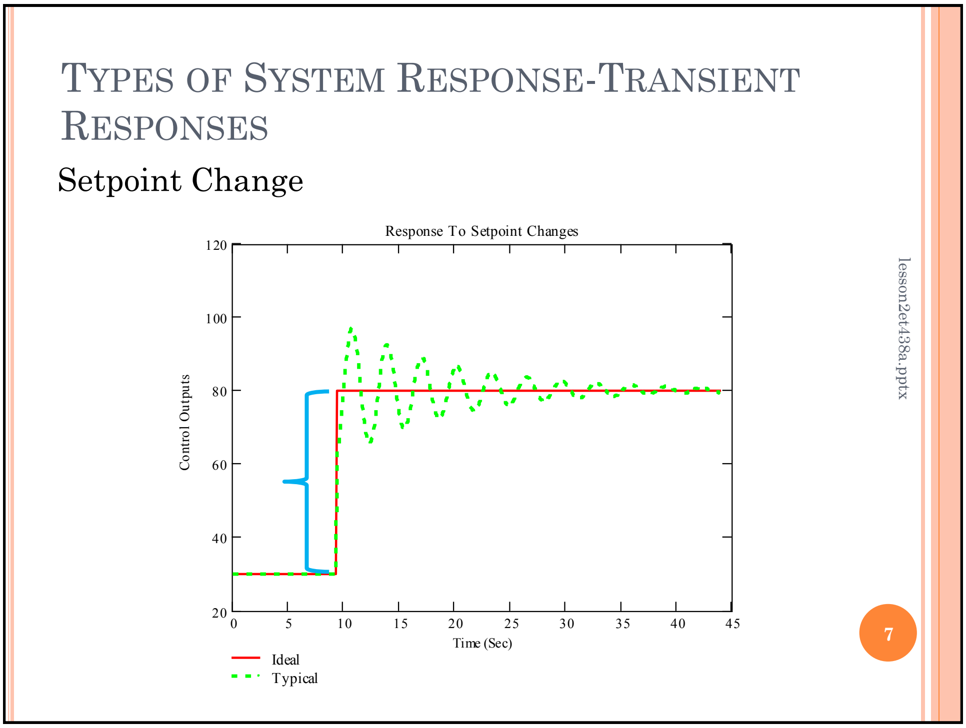
1.) Uncontrolled process
2.) Process control activated
3.) Unstable system



TYPES OF SYSTEM RESPONSE

Damped response

Control variable requires time to reach final value



TYPES OF SYSTEM RESPONSE-TRANSIENT RESPONSES

Setpoint Change

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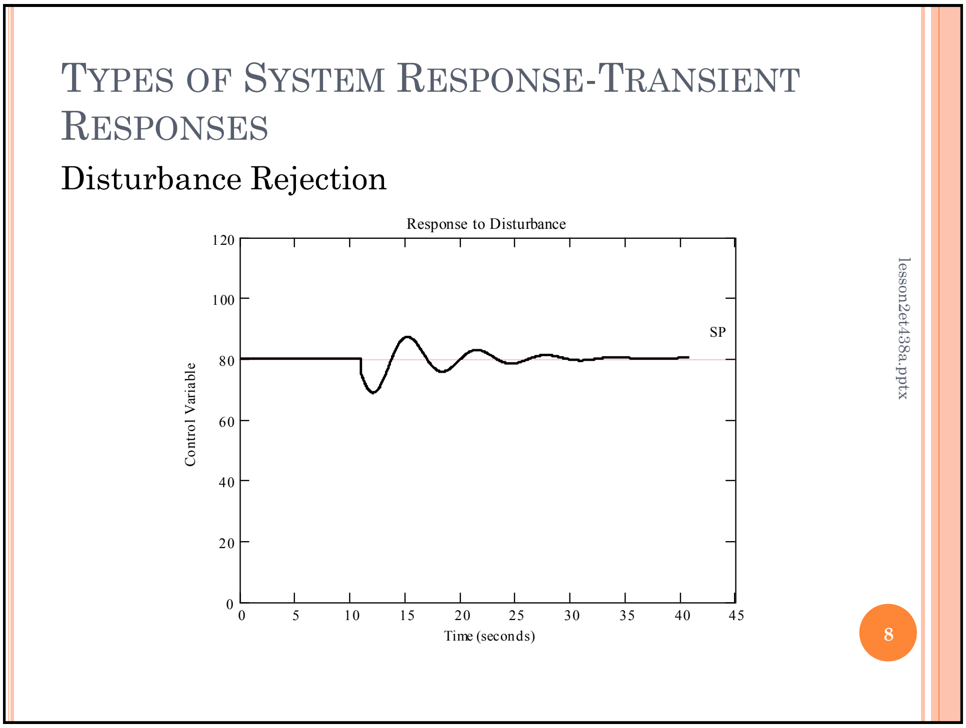

TYPES OF SYSTEM RESPONSE-TRANSIENT RESPONSES

Disturbance Rejection

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ANALOG MEASUREMENT ERRORS AND CONTROL SYSTEMS

Amount of error determines system accuracy

Determing accuracy

Percent of Full Scale (FS)
FS·(%/100)

Accuracy ± 5%
FS =10V
E=10V(± 5%/100)
E= ± 0.5 V

Percent of Span
Span=max-min
Span·(%/100)

Accuracy ± 3%
min=20, max= 50 psi
E=(50-20)(± 3%/100)
E= ± 0.9 psi

Percent of Reading
Reading·(%/100)

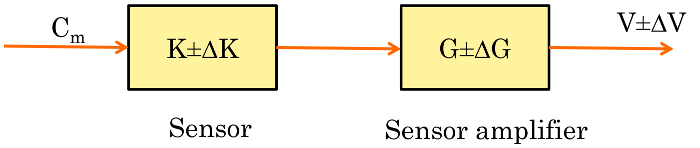
Accuracy ± 2%
Reading = 2 V
E=(2V)(± 2%/100)
E= ± 0.04 V

Measured Value
Reading ± Value

100 psi
± 2 psi

SYSTEM ACCURACY AND CUMULATIVE ERROR

Subsystem errors accumulate and determine accuracy limits.
Consider a measurement system



K = sensor gain
G = amplifier gain
V = sensor output voltage,
ΔG, ΔV, ΔK uncertainties in measurement

What is magnitude of ΔV?

SYSTEM ACCURACY AND CUMULATIVE ERROR

Output

Input

With no uncertainty: $V = K \cdot G \cdot C_m$

With uncertainty $V \pm \Delta V = (K \pm \Delta K) \cdot (G \pm \Delta G) \cdot C_m$

Multiple out and simplify to get:

$$\frac{\Delta V}{V} = \frac{\Delta G}{G} + \frac{\Delta K}{K}$$

Where :

$\frac{\Delta V}{V}$ = normalized uncertainty of output

Component Tolerance/100

$\frac{\Delta G}{G}$ = normalized uncertainty of sensor amp

$\frac{\Delta K}{K}$ = normalized uncertainty of sensor

COMBINING ERRORS

Use Root-Mean-Square (RMS) or Root Sum Square (RSS)

$$\frac{\Delta V}{V} = \sqrt{\left(\frac{\Delta G}{G}\right)^2 + \left(\frac{\Delta K}{K}\right)^2}$$

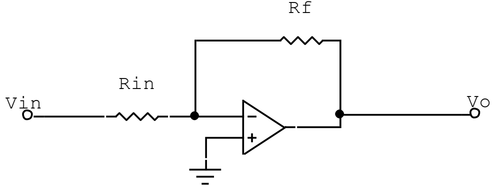
This relationship works on all formulas that include only multiplication and division.

Notes: $\frac{\Delta V}{V}$ is fractional RMS uncertainty. Multiply by 100 to get %

$\frac{\Delta G}{G}$, $\frac{\Delta K}{K}$ are fraction uncertainty. Divide tolerances by 100%

CUMULATIVE ERROR EXAMPLE

Example 2-1: Determine the RMS error (uncertainty) of the OP AMP circuit shown. The resistors R_f and R_{in} have tolerances of 5%. The input voltage V_{in} has a measurement tolerance of 2%.



$$V_o = V_{in} \cdot \left(\frac{-R_f}{R_{in}}\right)$$

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Determine uncertainty from tolerances

$$\left(\frac{\Delta R_f}{R_f}\right) = \left(\frac{\Delta R_{in}}{R_{in}}\right) = \frac{\pm 5\%}{100} = \pm 0.05$$

$$\left(\frac{\Delta V_{in}}{V_{in}}\right) = \frac{\pm 2\%}{100} = \pm 0.02$$

$$\left(\frac{\Delta V_o}{V_o}\right)_{RMS} = \pm\sqrt{\left(\frac{\Delta R_f}{R_f}\right)^2 + \left(\frac{\Delta R_{in}}{R_{in}}\right)^2 + \left(\frac{\Delta V_{in}}{V_{in}}\right)^2}$$

$$\left(\frac{\Delta V_o}{V_o}\right)_{RMS} = \pm\sqrt{0.05^2 + 0.05^2 + 0.02^2} = \pm 0.073$$

$$\%U = 100\% \cdot \left(\frac{\Delta V_o}{V_o}\right)_{RMS} = \pm 7.3\%$$

ANS

SENSOR CHARACTERISTICS

Sensitivity - Change in output for change in input. Equals the slope of I/O curve in linear device.

Hysteresis - output different for increasing or decreasing input.

Resolution - Smallest measurement a sensor can make.

Linearity - How close is the I/O relationship to a straight line.

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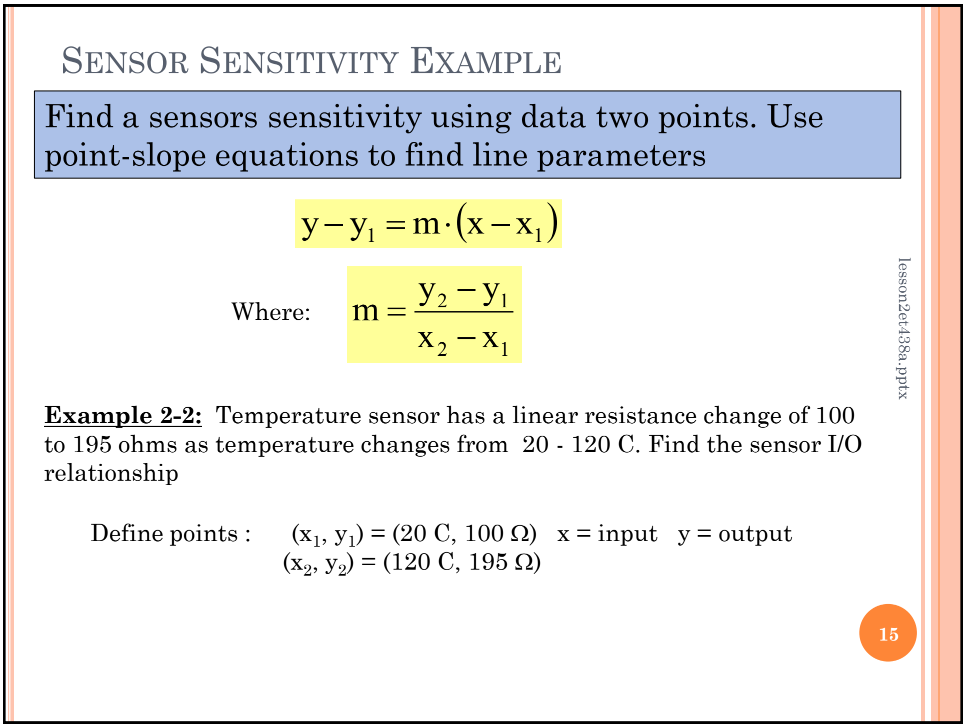
$$C_m = m \cdot C + C_0$$

Where C = control variable
m = slope
C_0 = offset (y intercept)
C_m = sensor output



SENSOR SENSITIVITY EXAMPLE

Find a sensors sensitivity using data two points. Use point-slope equations to find line parameters

$$y - y_1 = m \cdot (x - x_1)$$

Where: $$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Example 2-2: Temperature sensor has a linear resistance change of 100 to 195 ohms as temperature changes from 20 - 120 C. Find the sensor I/O relationship

Define points : $(x_1, y_1) = (20\ \text{C}, 100\ \Omega)$ x = input y = output
$(x_2, y_2) = (120\ \text{C}, 195\ \Omega)$

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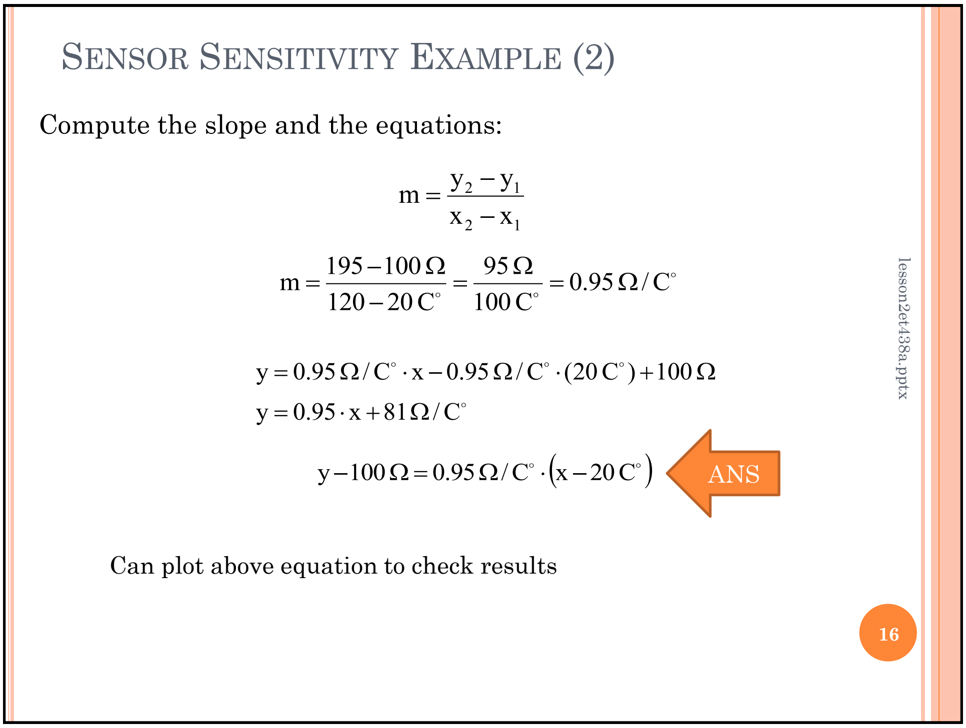

SENSOR SENSITIVITY EXAMPLE (2)

Compute the slope and the equations:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{195 - 100\,\Omega}{120 - 20\,C^\circ} = \frac{95\,\Omega}{100\,C^\circ} = 0.95\,\Omega / C^\circ$$

$$y = 0.95\,\Omega / C^\circ \cdot x - 0.95\,\Omega / C^\circ \cdot (20\,C^\circ) + 100\,\Omega$$
$$y = 0.95 \cdot x + 81\,\Omega / C^\circ$$

$$y - 100\,\Omega = 0.95\,\Omega / C^\circ \cdot (x - 20\,C^\circ)$$

ANS

Can plot above equation to check results

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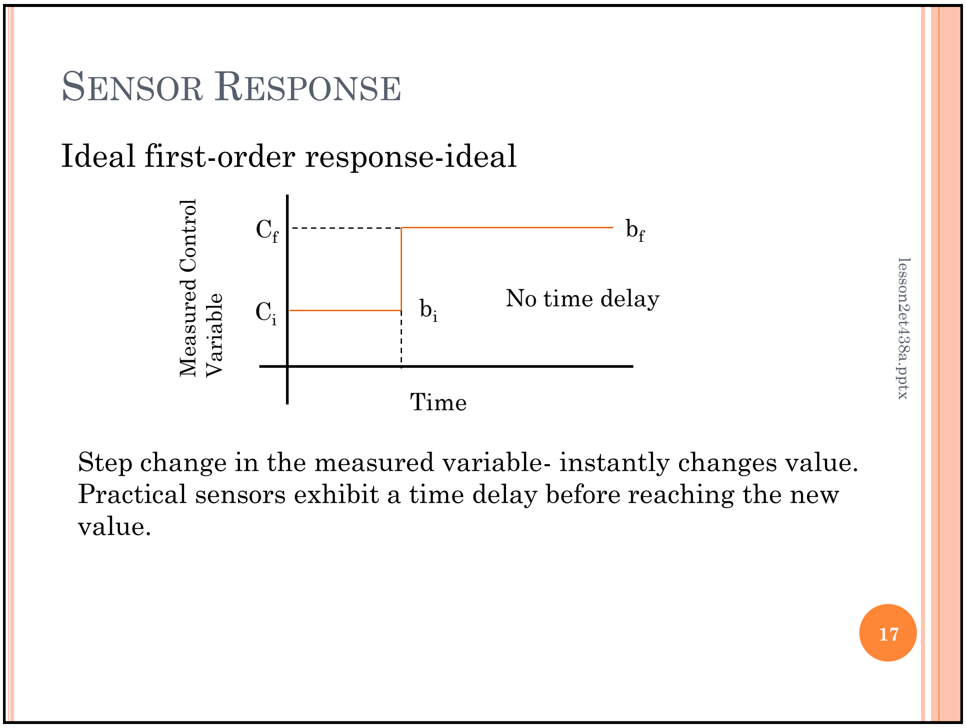
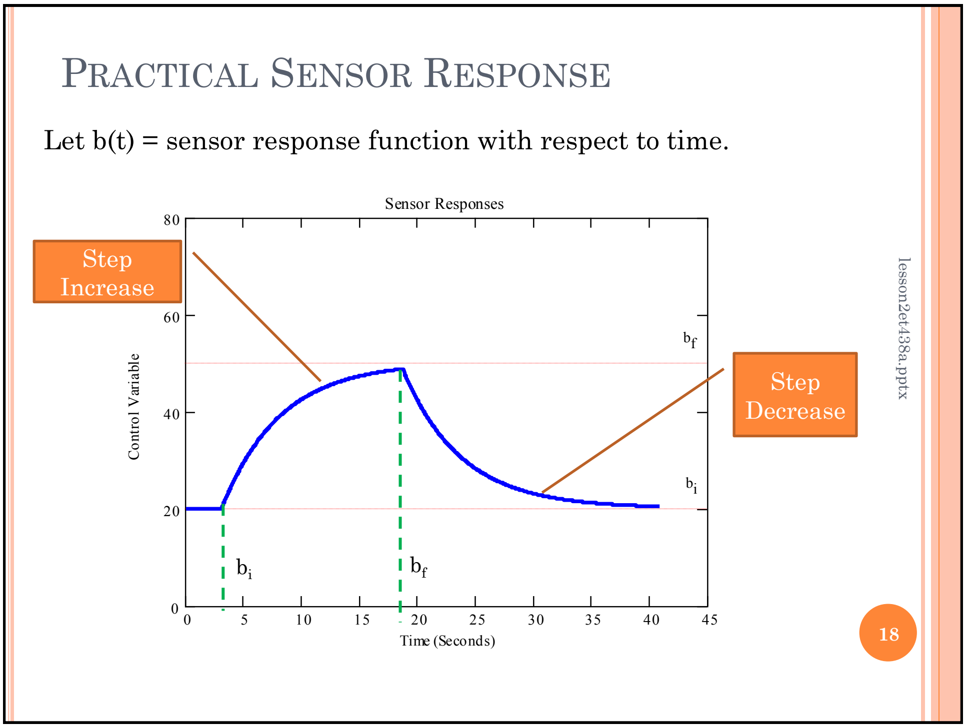


SENSOR RESPONSE

Ideal first-order response-ideal

Step change in the measured variable- instantly changes value. Practical sensors exhibit a time delay before reaching the new value.



PRACTICAL SENSOR RESPONSE

Let b(t) = sensor response function with respect to time.

MODELING 1ST-ORDER SENSOR RESPONSE

For step increase:

$$b(t) = b_i + (b_f - b_i)\cdot\left(1 - e^{\frac{-t}{\tau}}\right)$$

Where b_f = final sensor value
b_i = initial sensor value
t = time
τ = time constant of sensor

For step decrease:

$$b(t) = (b_i - b_f)\cdot\left(e^{\frac{-t}{\tau}}\right)$$

SENSOR RESPONSE EXAMPLE 1

Example 2-3: A control loop sensor detects a step increase and has an initial voltage output of b_i = 2.0 V Its final output is b_f = 4.0 V. It has a time constant of τ = 0.0025 /s. Find the time it takes to reach 90% of its final value.

$b_i = 2.0\text{V}$ $b_f = 4.0\text{V}$ $\tau = 0.0025\ /\text{s}$

$b(t) = b_i + (b_f - b_i)(1 - e^{-t/\tau})$ SET $b(t) = 90\%$ of b_f AND SOLVE FOR t

$$\left(\frac{90\%}{100\%}\right)(4.0) = 2.0 + (4.0 - 2.0)(1 - e^{-t/0.0025s})$$

$$3.6 = 2.0 + 2.0(1 - e^{-400t})$$

EXAMPLE 2-3 CONTINUED (2)

Complete the calculations to find t

$$3.6-2.0 = 2.0\left(1-e^{-400t}\right)$$
$$1.6 = 2.0\left(1-e^{-400t}\right)$$
$$0.8 = \left(1-e^{-400t}\right)$$
$$(0.8-1) = -e^{-400t}$$
$$-0.2 = -e^{-400t}$$

Take ln of both sides to solve for t $\ln(e^{x}) = x$

$$0.2 = e^{-400t}$$
$$Ln(0.2) = -400t$$
$$-1.6094 = -400t$$
$$1.6094 = 400t$$
$$\frac{1.6094}{400} = t \quad \boxed{0.00402\ sec}$$

Approx 4 ms

SENSOR RESPONSE EXAMPLE 2

Example 2-4: A sensor with a first order response characteristic has initial output of 1.0 V. How long does it take to decrease to 0.2 V if the time constant of the sensor is 0.1/s.

$b_i = 1.0V \quad b_f = 0.2 \quad \tau = 0.1 sec \quad b(t) = (b_i - b_f)\left(e^{-t/\tau}\right)$

Let $b(t) = 0.2V$
and solve for t

$$0.2V = (1.0V - 0.2V)\, e^{-t/0.1s}$$
$$0.2V = (0.8)\, e^{-10t}$$
$$\frac{0.2V}{0.8V} = e^{-10t} \qquad 0.25 = e^{-10t}$$

Take ln(x) of both sides
to solve for t
$Ln(e^{x}) = x$

$$Ln(0.25) = Ln\left(e^{-10t}\right)$$
$$-1.386 = -10t$$
$$\boxed{0.1386s = t}$$

SIGNIFICANT DIGITS IN INSTRUMENTATION AND CONTROL

Significant Digits In Measurement

Readable output of instruments
Resolution of sensors and transducers

Calculations Using Measurements

Truncate calculator answers to match significant digits of measurements and readings,

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SIGNIFICANT DIGIT EXAMPLES

Example 2-5: Compute power based on the following measured values. Use correct number of significant digits.

3.25 A	3 significant digits
117.8 V	4 significant digits

$$P = V \cdot I = (3.25\ \text{A}) \cdot (117.8\ \text{V}) = 382.85\ \text{W}$$

Truncate to 3 significant digits **P = 383 W**

Significant digits not factor in design calculations. Device values assumed to have no uncertainty.

SIGNIFICANT DIGIT EXAMPLES

Example 2-6: Compute the current flow through a resistor that has a measured R of 1.234 kΩ and a voltage drop of 1.344 Vdc.

R = 1.234 kΩ 4 significant digits
V = 1.344 V 4 significant digits

$I = (1.344)/(1.234\text{x } 10^3) = 1.089 \text{ mA}$ 4 digits

Since both measured values have four significant digits the computation result can have at most four significant digits.

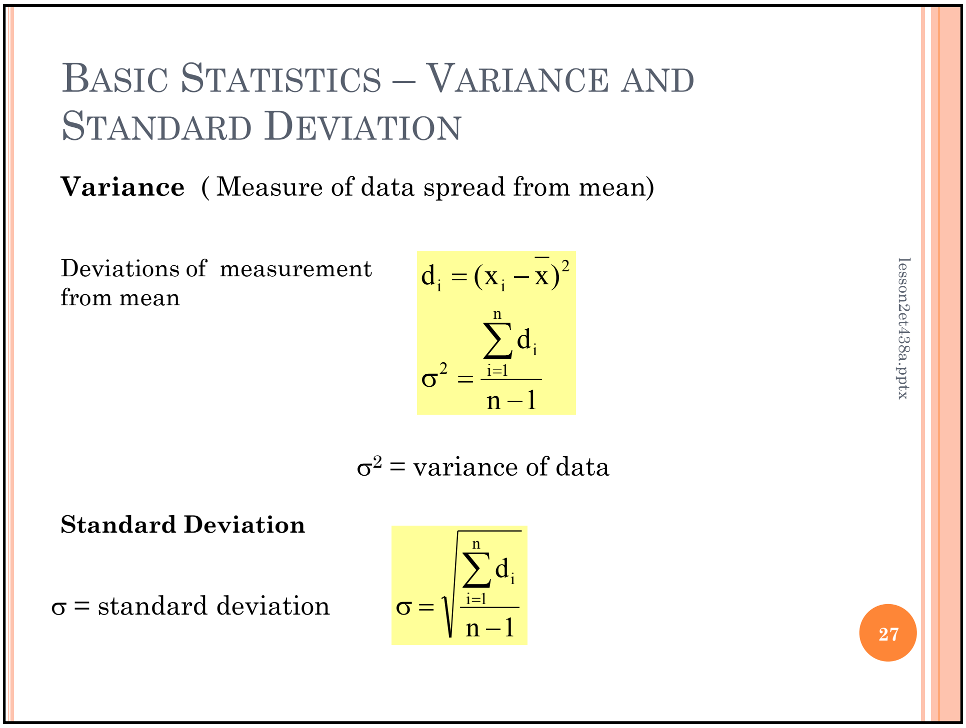
BASIC STATISTICS

Measurements can be evaluated using statistical measures such as mean variance and standard deviation.

Arithmetic Mean (Central Tendency)

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

Where x_i = i-th data measurement
n = total number of measurements taken



BASIC STATISTICS – VARIANCE AND STANDARD DEVIATION

Variance (Measure of data spread from mean)

Deviations of measurement from mean

$$d_i = (x_i - \bar{x})^2$$

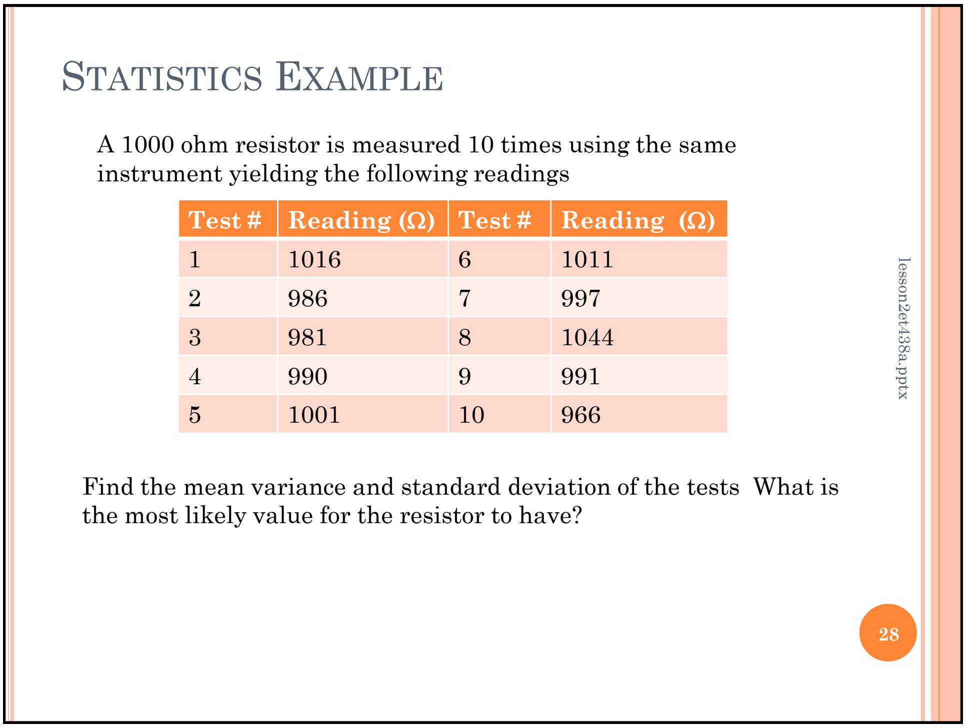
$$\sigma^2 = \frac{\sum_{i=1}^{n} d_i}{n-1}$$

σ^2 = variance of data

Standard Deviation

σ = standard deviation

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n} d_i}{n-1}}$$



STATISTICS EXAMPLE

A 1000 ohm resistor is measured 10 times using the same instrument yielding the following readings

Test #	Reading (Ω)	Test #	Reading (Ω)
1	1016	6	1011
2	986	7	997
3	981	8	1044
4	990	9	991
5	1001	10	966

Find the mean variance and standard deviation of the tests What is the most likely value for the resistor to have?

STATISTICS EXAMPLE SOLUTION

$$\bar{X} = \frac{1016+986+981+990+1001+1011+997+1044+991+966}{10}$$

$\boxed{\bar{X} = 998.3\,\Omega}$ MOST LIKELY VALUE FOR R IS $\bar{X}$ SO 998.3 Ω

Variance Calculations

$d_1 = (1016-998.3)^2$ $d_2 = (986-998.3)^2$

$d_1 = 313.29$ $d_2 = 151.3$

$$\sigma^2 = \frac{\sum_1^{10} d_i}{10-1} = \boxed{465.3}$$

Find Standard Deviation

$$\sigma = \sqrt{\sigma^2}$$

$$\sigma = \sqrt{465.3} = \boxed{21.6}$$

ALL MEMBERS OF SAMPLE ARE WITHIN ±3σ

END LESSON 2: PERFORMANCE OF CONTROL SYSTEMS

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